

**Mathematics 23B; Fall 2012; V. Ginzburg
Practice Midterm**

1. For each of the ten questions below, state whether the assertion is *true* or *false*. (You do not need to justify your answer.)

(a) The area of the region D is equal to $\iint_D dA$.

(b) The change of variable formula reads:

$$\iint_D f(x, y) dx dy = \iint_{D^*} f(x(u, v), y(u, v)) \frac{\partial(x, y)}{\partial(u, v)} du dv.$$

(c) The area of the region bounded by the curves $y = x^2 - 1$ and $y = 0$ is equal to $\int_{-1}^1 (x^2 - 1) dx$.

(d) Let ρ , ϕ , and θ be spherical coordinates. Then

$$\left| \frac{\partial(x, y, z)}{\partial(\rho, \theta, \phi)} \right| = \rho^2 \sin \phi.$$

(e) Every one-to-one map is necessarily onto.

(f) The Jacobian of the map $T(u, v) = (au + bv, cu + dv)$ is $ad + bc$.

(g) Let D be the region given by the inequalities $a \leq y \leq b$ and $\gamma_1(y) \leq x \leq \gamma_2(y)$. Then

$$\iint_D f(x, y) dx dy = \int_a^b \int_{\gamma_1(y)}^{\gamma_2(y)} f(x, y) dy dx.$$

(h) The average of the function $f(x, y, z)$ over the region W is

$$\frac{\iiint_W f(x, y, z) dV}{\iiint_W dV}.$$

(i) Every bounded function on the region D is integrable.

(j) $\iint_D x^7 \sin^2 y dx dy = 0$, where D is given by $x^2 + y^2 \leq 64$ and $y \geq 0$.

2. Evaluate the following double integrals:

(a) $\iint_D y^3 dx dy$, where the region D is given by the inequalities $0 \leq x \leq 1$ and $-e^x \leq y \leq e^x$.

(b) $\iint_D xy dA$, where the region D is bounded by $y = 2\sqrt{2}x^2$ and $y = \sqrt{x}$.

3. Evaluate the following triple integrals:

(a) $\iiint_W \frac{1}{xyz} dx dy dz$, where $W = [1, e] \times [1, e] \times [1, e]$.

(b) $\iiint_W z dV$, where W is the region in the first octant bounded by the cylinder $x^2 + y^2 = 9$ and the planes $y = z$, $x = 0$ and $z = 0$.

4. Evaluate

(a) $\iint_D \cos(x^2 + y^2) dx dy$, where the region D is given by the inequalities $0 \leq y \leq x$ and $x^2 + y^2 \leq \pi^2$.

(b) $\iiint_W e^{(x^2+y^2+z^2)^{\frac{3}{2}}} dV$, where W is the ball of radius 2 centered at the origin.

5. Find the volume of the solid that is bounded by the paraboloid $z = 9 - x^2 - y^2$, the xy -plane and the cylinder $x^2 + y^2 = 4$.