1. For each of the ten questions below, state whether the assertion is true or false. (You do not need to justify your answer.)

(a) The area of the portion of the graph of $z = f(x, y)$ over a region $D$ in the $(xy)$-plane is equal to

$$\iint_D \sqrt{1 + \left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2} \, dA.$$

(b) Let $\Omega$ be the region in $\mathbb{R}^3$ bounded by a surface $S$ and $\mathbf{F}$ a $C^1$ vector field in $\mathbb{R}^3$. Gauss’ theorem asserts that

$$\iiint_\Omega \nabla \cdot \mathbf{F} \, dV = \iint_S \mathbf{F} \cdot d\mathbf{S},$$

where $S$ is oriented inward.

(c) Let $S$ be the unit sphere. Then $\int_S x^2 \, dS = 0$.

(d) Let $z$, $r$, and $\theta$ be cylindrical coordinates. Then

$$\left| \frac{\partial (x, y, z)}{\partial (r, \theta, z)} \right| = r^2 \sin \theta.$$

(e) Every onto map is necessarily one-to-one.

(f) The integral $\int_S f \, dS$, where $f$ is a function, changes sign when the orientation of $S$ is changed.

(g) Let $D$ be the region to which Green’s theorem applies. Then the area of $D$ is equal to $\frac{1}{2} \int_{\partial D} (xdy - ydx)$.

(h) Let a surface $S$ be oriented by the unit normal $\mathbf{n}$. Then $\int_S \mathbf{F} \cdot d\mathbf{S} = \iint_S (\mathbf{F} \cdot \mathbf{n}) \, dS$, for any vector field $\mathbf{F}$.

(i) The vector field $\mathbf{F}(x, y) = \frac{-y + x^4}{x^2 + y^2}$ is conservative.

(j) $\nabla \cdot (\nabla \times \mathbf{F}) = 0$ for any $C^2$ vector field $\mathbf{F}$.

2. Evaluate the integral $\iint_D (\cos x - y) \, dx \, dy$, where the region $D$ is bounded by $y = \sin x$, $y = 0$, $x = 0$, and $x = \pi$.

3. Evaluate the integral $\iiint_W z \, dx \, dy \, dz$, where $W$ is the solid bounded by the paraboloid $z = x^2 + y^2$ and the plane $z = 4$. 
4. Find the area of the portion of the paraboloid \( z = 9 - x^2 - y^2 \) that lies over the \( z = 0 \) plane.

5. Find the volume of the solid inside the cylinders \( x^2 + y^2 = 1 \) and \( x^2 + z^2 = 1 \).

6. Evaluate the path integral \( \int_C f \, ds \), where \( f(x, y) = y \cos(2\pi x) \) and \( C \) is the triangle with sides \( x = 0 \), \( y = 0 \), and \( x + y = 1 \).

7. Evaluate the following line integrals:
   
   (a) \( \int_C \mathbf{F} \cdot ds \), where \( \mathbf{F}(x, y) = -2(y + x)\mathbf{i} + (3x + 2y)\mathbf{j} \) and \( C \) is the ellipse \( x^2/9 + y^2/3 = 1 \) oriented counter-clockwise.
   
   (b) \( \int_C (x \, dx + y \, dy + z^2 \, dz) \), where \( C \) is the curve parametrized as \( C(t) = (\cos t, \sin t, t) \) with \( 0 \leq t \leq 1 \).

8. Evaluate the surface integral
   
   \[ \iint_S \mathbf{F} \cdot d\mathbf{S} , \]
   
   where \( \mathbf{F}(x, y, z) = x\mathbf{i} + y\mathbf{j} + \mathbf{k} \) and \( S \) is the upper hemisphere \( x^2 + y^2 + z^2 = 1 \), \( z \geq 0 \).

9. Let \( F(z, y, z) = yz^2\mathbf{i} + xz^2\mathbf{j} + 2xyz\mathbf{k} \).
   
   (a) Evaluate \( \nabla \times \mathbf{F} \). Is \( \mathbf{F} \) conservative?
   
   (b) Find a function \( f \) such that \( \mathbf{F} = \nabla f \).

10. Use Stokes’ theorem to evaluate the integral \( \int_C \mathbf{F} \cdot d\mathbf{S} \), where \( \mathbf{F}(x, y, z) = -xy\mathbf{i} - xz\mathbf{j} - yz\mathbf{k} \) and \( C \) is the triangle with vertices \((0, 1, 0)\), \((0, 1, 5)\), and \((3, 1, 0)\) oriented by the ordering of the points.

11. Use Gauss’ theorem to evaluate the surface integral \( \iint_S \mathbf{F} \cdot d\mathbf{S} \), where \( \mathbf{F}(x, y, z) = x^2\mathbf{i} + y^2\mathbf{j} + z^2\mathbf{k} \) and \( S \) is the cylinder bounded by \( x^2 + y^2 = 4 \) and the planes \( z = 0 \) and \( z = 1 \) (the top and the bottom are included), oriented outward.