Mathematics 23B; Fall 2010; V. Ginzburg
Practice Midterm

1. For each of the ten questions below, state whether the assertion is *true* or *false*. (You do not need to justify your answer.)

   (a) The area of the region D is equal to $\iint_D dA$.

   (b) The change of variable formula reads:

   $$\iint_D f(x,y)\,dx\,dy = \iint_{D^*} f(x(u,v),y(u,v))\frac{\partial(x,y)}{\partial(u,v)}\,du\,dv.$$

   (c) The area of the region bounded by the curves $y = x^2 - 1$ and $y = 0$ is equal to $\int_{-1}^{1}(x^2-1)\,dx$.

   (d) Let ρ, ϕ, and θ be spherical coordinates. Then

   $$\left|\frac{\partial(x,y,z)}{\partial(\rho,\theta,\phi)}\right| = \rho^2 \sin\phi.$$

   (e) Every one-to-one map is necessarily onto.

   (f) The Jacobian of the map $T(u,v) = (au+bv, cu+dv)$ is $ad+bc$.

   (g) Let D be the region given by the inequalities $a \le y \le b$ and $\gamma_1(y) \le x \le \gamma_2(y)$. Then

   $$\iint_D f(x,y)\,dx\,dy = \int_a^b \int_{\gamma_1(y)}^{\gamma_2(y)} f(x,y)\,dy\,dx.$$

   (h) The average of the function $f(x,y,z)$ over the region W is

   $$\frac{\iiint_W f(x,y,z)\,dV}{\iiint_W dV}.$$

   (i) Every bounded function on the region D is integrable.

   (j) $\iint_D x^7 \sin^2 y\,dx\,dy = 0$, where D is given by $x^2+y^2 \le 64$ and $y \ge 0$.

2. Evaluate the following double integrals:

   (a) $\iint_D y^3\,dx\,dy$, where the region D is given by the inequalities $0 \le x \le 1$ and $-e^x \le y \le e^x$.

   (b) $\iint_D xy\,dA$, where the region D is bounded by $y = 2\sqrt{2}x^2$ and $y = \sqrt{x}$.

3. Evaluate the following triple integrals:

   (a) $\iiint_W \frac{1}{xyz}\, dx\, dy\, dz$, where $W = [1, e] \times [1, e] \times [1, e]$.

   (b) $\iiint_W z\, dV$, where W is the region in the first octant bounded by the cylinder $x^2 + y^2 = 9$ and the planes $y = z$, $x = 0$ and $z = 0$.

4. Evaluate

   (a) $\iint_D \cos(x^2 + y^2)\, dx\, dy$, where the region D is given by the inequalities $0 \leq y \leq x$ and $x^2 + y^2 \leq \pi^2$.

   (b) $\iiint_W e^{(x^2+y^2+z^2)^{\frac{3}{2}}}\, dV$, where W is the ball of radius 2 centered at the origin.

5. Find the volume of the solid that is bounded by the paraboloid $z = 9 - x^2 - y^2$, the xy-plane and the cylinder $x^2 + y^2 = 4$.