

**Mathematics 23B; Fall 2009; V. Ginzburg
Practice Final**

1. For each of the ten questions below, state whether the assertion is *true* or *false*. (You do not need to justify your answer.)

- (a) The area of the portion of the graph of $z = f(x, y)$ over a region D in the (xy) -plane is equal to

$$\iint_D \sqrt{1 + \left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2} dA.$$

- (b) Let Ω be the region in \mathbb{R}^3 bounded by a surface S and \mathbf{F} a C^1 vector field in \mathbb{R}^3 . Gauss' theorem asserts that

$$\iiint_{\Omega} \nabla \cdot \mathbf{F} dV = \iint_S \mathbf{F} \cdot d\mathbf{S},$$

where S is oriented inward.

- (c) Let S be the unit sphere. Then $\iint_S x^2 z dS = 0$.
 (d) Let z , r , and θ be cylindrical coordinates. Then

$$\left| \frac{\partial(x, y, z)}{\partial(r, \theta, z)} \right| = r^2 \sin \theta.$$

- (e) Every onto map is necessarily one-to-one.
 (f) The integral $\iint_S f dS$, where f is a function, changes sign when the orientation of S is changed.
 (g) Let D be the region to which Green's theorem applies. Then the area of D is equal to $\frac{1}{2} \int_{\partial D} (xdy - ydx)$.
 (h) Let a surface S be oriented by the unit normal \mathbf{n} . Then $\iint_S \mathbf{F} \cdot d\mathbf{S} = \iint_S (\mathbf{F} \cdot \mathbf{n}) dS$, for any vector field \mathbf{F} .
 (i) The vector field $\mathbf{F}(x, y) = \frac{-y\mathbf{i} + x\mathbf{j}}{x^2 + y^2}$ is conservative.
 (j) $\nabla \cdot (\nabla \times \mathbf{F}) = 0$ for any C^2 vector field \mathbf{F} .
2. Evaluate the integral $\iint_D (\cos x - y) dx dy$, where the region D is bounded by $y = \sin x$, $y = 0$, $x = 0$, and $x = \pi$.
3. Evaluate the integral $\iiint_W z dx dy dz$, where W is the solid bounded by the paraboloid $z = x^2 + y^2$ and the plane $z = 4$.

4. Find the area of the portion of the paraboloid $z = 9 - x^2 - y^2$ that lies over the $z = 0$ plane.
5. Find the volume of the solid inside the cylinders $x^2 + y^2 = 1$ and $x^2 + z^2 = 1$.
6. Evaluate the path integral $\int_C f ds$, where $f(x, y) = y \cos(2\pi x)$ and C is the triangle with sides $x = 0$, $y = 0$, and $x + y = 1$.
7. Evaluate the following line integrals:
 - (a) $\int_C \mathbf{F} \cdot d\mathbf{s}$, where $\mathbf{F}(x, y) = -2(y + x)\mathbf{i} + (3x + 2y)\mathbf{j}$ and C is the ellipse $x^2/9 + y^2/3 = 1$ oriented counter-clockwise.
 - (b) $\int_C (x dx + y dy + z^2 dz)$, where C is the curve parametrized as $C(t) = (\cos t, \sin t, t)$ with $0 \leq t \leq 1$.

8. Evaluate the surface integral

$$\iint_S \mathbf{F} \cdot d\mathbf{S},$$

where $\mathbf{F}(x, y, z) = x\mathbf{i} + y\mathbf{j} + \mathbf{k}$ and S is the upper hemisphere $x^2 + y^2 + z^2 = 1$, $z \geq 0$.

9. Let $F(z, y, z) = yz^2\mathbf{i} + xz^2\mathbf{j} + 2xyz\mathbf{k}$.
 - (a) Evaluate $\nabla \times \mathbf{F}$. Is \mathbf{F} conservative?
 - (b) Find a function f such that $\mathbf{F} = \nabla f$.
10. Use Stokes' theorem to evaluate the integral $\int_C \mathbf{F} \cdot d\mathbf{S}$, where $\mathbf{F}(x, y, z) = -xy\mathbf{i} - xz\mathbf{j} - yz\mathbf{k}$ and C is the triangle with vertices $(0, 1, 0)$, $(0, 1, 5)$, and $(3, 1, 0)$ oriented by the ordering of the points.
11. Use Gauss' theorem to evaluate the surface integral $\iint_S \mathbf{F} \cdot d\mathbf{S}$, where $\mathbf{F}(x, y, z) = x^2\mathbf{i} + y^2\mathbf{j} + z^2\mathbf{k}$ and S is the cylinder bounded by $x^2 + y^2 = 4$ and the planes $z = 0$ and $z = 1$ (the top and the bottom are included), oriented outward.