

**Mathematics 23B; Fall 2008; V. Ginzburg  
Practice Midterm**

1. For each of the ten questions below, state whether the assertion is *true* or *false*. (You do not need to justify your answer.)

(a) The area of the region  $D$  is equal to  $\iint_D dA$ .

(b) The change of variable formula reads:

$$\iint_D f(x, y) dx dy = \iint_{D^*} f(x(u, v), y(u, v)) \frac{\partial(x, y)}{\partial(u, v)} du dv.$$

(c) The area of the region bounded by the curves  $y = x^2 - 1$  and  $y = 0$  is equal to  $\int_{-1}^1 (x^2 - 1) dx$ .

(d) Let  $\rho$ ,  $\phi$ , and  $\theta$  be spherical coordinates. Then

$$\left| \frac{\partial(x, y, z)}{\partial(\rho, \theta, \phi)} \right| = \rho^2 \sin \phi.$$

(e) Every one-to-one map is necessarily onto.

(f) The Jacobian of the map  $T(u, v) = (au + bv, cu + dv)$  is  $ad + bc$ .

(g) Let  $D$  be the region given by the inequalities  $a \leq y \leq b$  and  $\gamma_1(y) \leq x \leq \gamma_2(y)$ . Then

$$\iint_D f(x, y) dx dy = \int_a^b \int_{\gamma_1(y)}^{\gamma_2(y)} f(x, y) dy dx.$$

(h) The average of the function  $f(x, y, z)$  over the region  $W$  is

$$\frac{\iiint_W f(x, y, z) dV}{\iiint_W dV}.$$

(i) Every bounded function on the region  $D$  is integrable.

(j)  $\iint_D x^7 \sin^2 y dx dy = 0$ , where  $D$  is given by  $x^2 + y^2 \leq 64$  and  $y \geq 0$ .

2. Evaluate the following double integrals:

(a)  $\iint_D y^3 dx dy$ , where the region  $D$  is given by the inequalities  $0 \leq x \leq 1$  and  $-e^x \leq y \leq e^x$ .

(b)  $\iint_D xy dA$ , where the region  $D$  is bounded by  $y = 2\sqrt{2}x^2$  and  $y = \sqrt{x}$ .

3. Evaluate the following triple integrals:

(a)  $\iiint_W \frac{1}{xyz} dx dy dz$ , where  $W = [1, e] \times [1, e] \times [1, e]$ .

(b)  $\iiint_W z dV$ , where  $W$  is the region in the first octant bounded by the cylinder  $x^2 + y^2 = 9$  and the planes  $y = z$ ,  $x = 0$  and  $z = 0$ .

4. Evaluate

(a)  $\iint_D \cos(x^2 + y^2) dx dy$ , where the region  $D$  is given by the inequalities  $0 \leq y \leq x$  and  $x^2 + y^2 \leq \pi^2$ .

(b)  $\iiint_W e^{(x^2+y^2+z^2)^{\frac{3}{2}}} dV$ , where  $W$  is the ball of radius 2 centered at the origin.

5. Find the volume of the solid that is bounded by the paraboloid  $z = 9 - x^2 - y^2$ , the  $xy$ -plane and the cylinder  $x^2 + y^2 = 4$ .