

**Mathematics 23B; Spring 2004; V. Ginzburg  
Practice Final**

1. For each of the ten questions below, state whether the assertion is *true* or *false*. (You do not need to justify your answer.)

- (a) The area of the portion of the graph of  $z = f(x, y)$  over a region  $D$  in the  $(xy)$ -plane is equal to

$$\iint_D \sqrt{1 + \left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2} dA.$$

- (b) Let  $\Omega$  be the region in  $\mathbb{R}^3$  bounded by a surface  $S$  and  $\mathbf{F}$  a  $C^1$  vector field in  $\mathbb{R}^3$ . Gauss' theorem asserts that

$$\iiint_{\Omega} \nabla \cdot \mathbf{F} dV = \iint_S \mathbf{F} \cdot d\mathbf{S},$$

where  $S$  is oriented inward.

- (c) Let  $S$  be the unit sphere. Then  $\iint_S x^2 z dS = 0$ .  
 (d) Let  $z$ ,  $r$ , and  $\theta$  be cylindrical coordinates. Then

$$\left| \frac{\partial(x, y, z)}{\partial(r, \theta, z)} \right| = r^2 \sin \theta.$$

- (e) Every onto map is necessarily one-to-one.  
 (f) The integral  $\iint_S f dS$ , where  $f$  is a function, changes sign when the orientation of  $S$  is changed.  
 (g) Let  $D$  be the region to which Green's theorem applies. Then the area of  $D$  is equal to  $\frac{1}{2} \int_{\partial D} (x dy - y dx)$ .  
 (h) Let a surface  $S$  be oriented by the unit normal  $\mathbf{n}$ . Then  $\iint_S \mathbf{F} \cdot d\mathbf{S} = \iint_S (\mathbf{F} \cdot \mathbf{n}) dS$ , for any vector field  $\mathbf{F}$ .  
 (i) The vector field  $\mathbf{F}(x, y) = \frac{-y\mathbf{i} + x\mathbf{j}}{x^2 + y^2}$  is conservative.  
 (j)  $\nabla \cdot (\nabla \times \mathbf{F}) = 0$  for any  $C^2$  vector field  $\mathbf{F}$ .

2. Evaluate the integral  $\iint_D (\cos x - y) dx dy$ , where the region  $D$  is bounded by  $y = \sin x$ ,  $y = 0$ ,  $x = 0$ , and  $x = \pi$ .

3. Evaluate the integral  $\iiint_W z dx dy dz$ , where  $W$  is the solid bounded by the paraboloid  $z = x^2 + y^2$  and the plane  $z = 4$ .

4. Find the area of the portion of the paraboloid  $z = 9 - x^2 - y^2$  that lies over the  $z = 0$  plane.
5. Find the volume of the solid inside the cylinders  $x^2 + y^2 = 1$  and  $x^2 + z^2 = 1$ .
6. Evaluate the path integral  $\int_C f ds$ , where  $f(x, y) = y \cos(2\pi x)$  and  $C$  is the triangle with sides  $x = 0$ ,  $y = 0$ , and  $x + y = 1$ .
7. Evaluate the following line integrals:
  - (a)  $\int_C \mathbf{F} \cdot d\mathbf{s}$ , where  $\mathbf{F}(x, y) = -2(y + x)\mathbf{i} + (3x + 2y)\mathbf{j}$  and  $C$  is the ellipse  $x^2/9 + y^2/3 = 1$  oriented counter-clockwise.
  - (b)  $\int_C (x dx + y dy + z^2 dz)$ , where  $C$  is the curve parametrized as  $C(t) = (\cos t, \sin t, t)$  with  $0 \leq t \leq 1$ .

8. Evaluate the surface integral

$$\iint_S \mathbf{F} \cdot d\mathbf{S},$$

where  $\mathbf{F}(x, y, z) = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$  and  $S$  is the upper hemisphere  $x^2 + y^2 + z^2 = 1$ ,  $z \geq 0$ .

9. Let  $F(z, y, z) = yz^2\mathbf{i} + xz^2\mathbf{j} + 2xyz\mathbf{k}$ .
  - (a) Evaluate  $\nabla \times \mathbf{F}$ . Is  $\mathbf{F}$  conservative?
  - (b) Find a function  $f$  such that  $\mathbf{F} = \nabla f$ .
10. Use Stokes' theorem to evaluate the integral  $\int_C \mathbf{F} \cdot d\mathbf{S}$ , where  $\mathbf{F}(x, y, z) = -xy\mathbf{i} - xz\mathbf{j} - yz\mathbf{k}$  and  $C$  is the triangle with vertices  $(0, 1, 0)$ ,  $(0, 1, 5)$ , and  $(3, 1, 0)$  oriented by the ordering of the points.
11. Use Gauss' theorem to evaluate the surface integral  $\iint_S \mathbf{F} \cdot d\mathbf{S}$ , where  $\mathbf{F}(x, y, z) = x^2\mathbf{i} + y^2\mathbf{j} + z^2\mathbf{k}$  and  $S$  is the cylinder bounded by  $x^2 + y^2 = 4$  and the planes  $z = 0$  and  $z = 1$  (the top and the bottom are included), oriented outward.