MIDTERM PRACTICE PROBLEMS, MATH 23A

(1) Consider the following vectors: $\vec{a} = (-1, 0, 3)$, $\vec{b} = (2, 1, 0)$, $\vec{c} = (5, 1, -2)$ and $\vec{d} = (-2/\sqrt{5}, -1/\sqrt{5}, 0)$. In questions (a)-(c), indicate if the statement is true or false; you do **not** need to justify your answer.
   (a) The vector $\vec{a}$ is parallel to $\vec{d}$.
   (b) The vector $\vec{a} + \vec{b}$ is perpendicular to $\vec{c}$.
   (c) The vector $\vec{d}$ is a unit vector in the direction of $\vec{b}$.

(2) Let $\vec{a}$, $\vec{b}$, and $\vec{c}$ be as above.
   (a) Compute $a \times b$ and $\| \vec{a} - \vec{b} \|$.
   (a) Find a unit vector perpendicular to both $\vec{a}$ and $\vec{b}$.
   (b) Find the area of the parallelogram spanned by $\vec{a}$ and $\vec{c}$.
   (c) Find the volume of the parallelepiped spanned by $\vec{a}$, $\vec{b}$, and $\vec{c}$.

(3) Evaluate
$$\begin{vmatrix} 1 & 2 & 3 \\ 0 & -1 & 2 \\ 4 & 5 & 1 \end{vmatrix}.$$

(4) Find the equation of the line through $(-1, 1, 0)$ and $(2, 3, 4)$

(5) Show that the lines $x = 4 - t$, $y = 2t$, $z = 1 + t$ and $x = 18 + 2t$, $y = -1 - 4t$, $z = 1 - 2t$ are parallel.

(6) The planes $-y + 2z = 7$ and $x + z = 1$ intersect along a line.
   (a) Find the equation of this line.
   (b) Determine if this line intersect the line $x = t$, $y = 2t - 1$, $z = 0$.
   (c) Find the equation of the plane that contains the point $P = (1, -1, 0)$ and is perpendicular to this line.

(7) Describe the solid region bounded by the unit sphere $x^2 + y^2 + z^2 = 1$ and the elliptic cone $z = \sqrt{x^2 + y^2}$ in $\mathbb{R}^3$ using spherical coordinates, i.e., give the ranges of the coordinates ρ, θ and ϕ.

(8) Find the distance from the point $(1, 1, 1)$ to the plane $4x + 3z + 5 = 0$.

(9) Sketch the level surfaces and the graph of $f : \mathbb{R}^2 \to \mathbb{R}$ defined as $f(x, y) = -xy$.

(10) Find the domain and the range of the function $f(x, y) = \sqrt{4 - x^2 - y^2}$. Sketch the level surfaces and the graph of this function.

(11) Consider a $f : \mathbb{R}^3 \to \mathbb{R}$ defined by $f(x, y, z) = 4z^2 + x^2$.
   (a) Find the domain and the range of f.
   (b) Describe the level surfaces of the function f corresponding to the constant values $c > 0$, $c = 0$ and $c < 0$.
   (c) Let g be the restriction of f to the xy-plane, i.e, $g(x, y) = f(x, y, 0)$. Describe the level curves of the function g for the constant values $c > 0$, $c = 0$ and $c < 0$.

(12) Describe (sketch) the surface in $\mathbb{R}^3$ determined by the equation:
 (a) $y^2 = x - 1$
 (b) $z^2 = x^2 + y^2$
 (b) $z = x^2 + y^2$

(13) Evaluate the following limits or explain why a limit fails to exist:
 (a) $\lim_{(x,y)\to(0,0)} \dfrac{xy}{x^2+y^2+1}$
 (b) $\lim_{(x,y)\to(0,0)} \dfrac{e^{xy}-1}{y}$
 (c) $\lim_{(x,y)\to(1,0)} \dfrac{\sin y}{1-x}$

(14) Is the function defined as $\lim_{(x,y)\to(0,0)} \dfrac{y}{x^2+y^2}$ for $(x,y) \neq (0,0)$ and $f(0,0) = 0$ continuous at $(0,0)$? Justify your answer.

(15) Find $\partial f/\partial x$ and $\partial f/\partial y$ if $f(u,v) = \frac{u^2+v^2}{u^2-v^2}$, where $u(x,y) = e^{-x-y}$ and $v(x,y) = e^{xy}$.

(16) Find all first order partial derivatives of the following functions:
 (a) $f(x,y,z) = \tan(x^3 + 3zy)$
 (b) $f(p,q) = e^{-p+q^4} + pq + 9$

(17) Use the chain rule to compute the derivative of f with respect to t in (a) and (b):
 (a) $f(x,y) = \ln(x + y^2)$, where $x(t) = 3t^2$ and $y(t) = e^{-t}$
 (b) $f(x,y) = y \cos x$, where $x(t) = e^t$ and $y(t) = \cos t$

(18) Let $f(x,y,z) = xe^z + y^2 - 4$ and $x(t,s) = t^2 - s$, $y(t,s) = \cos t$, $z(t,s) = \ln(st)$. Use the chain rule to compute $\partial f/\partial t$ and $\partial f/\partial s$.

(19) Compute the second order partial derivatives $\partial^2 f/\partial x^2$, $\partial^2 f/\partial x\,\partial y$, $\partial^2 f/\partial y\,\partial x$ and $\partial^2 f/\partial y^2$ for the following functions:
 (a) $f(x,y) = \cos(xy^2)$
 (b) $f(x,y) = e^{-xy^2} + y^3x^4$

(20) Determine the velocity and acceleration vectors and the equation of the tangent line for each of the following curves at the specified value of t:
 (a) $\gamma(t) = (\sin(3t), \cos(3t), 2t^{3/2})$; $t = \pi$
 (b) $\gamma(t) = (\cos^2 t, 3t - t^3, t)$; $t = 0$

(21) Let $f(x,y) = \frac{e^z}{x^2+y^2}$.
 (a) Compute ∇f.
 (b) Compute the directional derivative of f at $(1,1,1)$ in the direction of the vector $\vec{n} = (\vec{i} - \vec{j})/\sqrt{2}$.

(22) Find the gradient and directional derivative of the function $f(x,y) = \ln(x^2 + y^2)$ at the point $(-1,1)$ in the direction of the vector $-\vec{j}$.

(23) Find the equation of the tangent plane to the surface determined by the equation $y^3x - xz^2 + z^5 = 9$ at the point $P = (-1,3,2)$. Find a unit normal vector to this surface at P. There are two such vectors. Why?

(24) If the vector $\vec{v} = (a^2, -2a, -1)$ lies in the tangent plane to the surface $z = e^x/y$ at the point $(0,1,1)$, what is the value of the constant a?