MATH 23A, REVIEW PROBLEMS FOR THE FINAL

(1) Find the equation of the tangent plane to the surface determined by the equation $y^3x - xz^2 + z^5 = 9$ at the point $P = (-1, 3, 2)$.

(2) If the vector $\mathbf{v} = (a^2, -2a, -1)$ lies in the tangent plane to the surface $z = e^x/y$ at the point $(0, 1, 1)$, what is the value of the constant a?

(3) Find the directional derivative of the function $f(x, y) = \ln(x^2 + y^2)$ at the point $(-1, 1)$ along the direction of the vector $\mathbf{v} = (3\mathbf{i} + 4\mathbf{j})/5$.

(4) Find Df and the Hessian matrix Hf for the following functions at the given points. Find also the second order Taylor formula for these functions at the given points.
   (a) $f(x, y) = (x - y)^2$; $(0, 0)$
   (b) $f(x, y) = e^{(y+3)^2} \cos x$; $(0, -3)$
   (c) $f(x, y) = \frac{\sin x}{y}$; $(\pi/2, 1)$

(5) Find the critical points of the functions (a)-(e), and classify them.
   (a) $f(x, y) = 3y^2 + 2xy + 2y + x^2 + x + 4$
   (b) $f(x, y) = e^{1+x^2-y^2}$
   (c) $f(x, y) = (x - y)(xy - 1)$
   (d) $f(x, y) = x^3 + y^2 - 6xy + 6x + 3y$
   (e) $f(x, y) = x \sin y$

(6) Determine whether $f(x, y) = x^4y^4$ has a local and an absolute maximum, a minimum, or neither.

(7) Find the absolute maximum and minimum values for $f(x, y) = \cos x + \sin y$ on the rectangle $R = [0, 2\pi] \times [0, 2\pi]$.

(8) Find the point in the plane $2x - y + 2z = 20$ nearest the origin.

(9) Find the points on the ellipse $x^2 + 2y^2 = 1$ where $f(x, y) = xy$ has its extreme values.

(10) Find the absolute maximum and minimum values for $f(x, y) = x^2 + xy + y^2$ on the unit disk $D = \{(x, y) | x^2 + y^2 \leq 1\}$. Use the method of Lagrange multipliers on the boundary.

(11) Find the absolute maximum and minimum values for $f(x, y) = x^2 + 3y^2 + 2y$ on the unit disk $D = \{(x, y) | x^2 + y^2 \leq 1\}$. Use the method of Lagrange multipliers on the boundary.

(12) Find the arc-length of the graph of $y = \cosh x$ for $-\ln 2 \leq x \leq \ln 2$.

(13) Find the arc-length of the path $\mathbf{c}(t) = (2\cos t, 2\sin t, t)$ for $0 \leq t \leq 2\pi$.

(14) Show that the equation $x^4yz + y^4 + z^4 = 1$ is solvable for z as a function of (x, y) near $(1, 1, 0)$. Compute $\partial z/\partial x$ and $\partial z/\partial y$ at $(1, 1)$.

(15) Compute the divergence and curl of the following vector fields $\mathbf{F}$, i.e., find $\nabla \cdot \mathbf{F}$ and $\nabla \times \mathbf{F}$.
  (a) $\mathbf{F}(x, y, z) = x(\sin y)\mathbf{i} + \cos(x - z)\mathbf{j} + z\mathbf{k}$
  (b) $\mathbf{F}(x, y, z) = (x - y)\mathbf{i} + (x + y)\mathbf{j}$
  (c) $\mathbf{F}(x, y, z) = \ln(z + y^2)\mathbf{i} + e^{x-y}\mathbf{j} + z^3 y\mathbf{k}$

(16) Show that $\mathbf{F}(x, y) = (x - y)\mathbf{i} + (y - x)\mathbf{j}$ is a gradient vector field. Find a function f such that $\mathbf{F} = \nabla f$.

(17) Let $\mathbf{F}(x, y) = x\mathbf{i} + x^2\mathbf{j}$ be the velocity field of a fluid in the plane.
  (a) Show that $\mathbf{c}(t) = (e^t, e^{2t}/2)$ is a flow line of $\mathbf{F}$.
  (b) Is $\mathbf{F}$ a gradient vector field? Justify your answer.
  (c) Is the fluid expanding or compressing as it moves? Justify your answer.

(18) Let $\mathbf{F}(x, y) = y\mathbf{i} + x\mathbf{j}$ be the velocity field of a fluid in the plane.
  (a) Show that $\mathbf{c}(t) = (2e^t - e^{-t}, 2e^t + e^{-t})$ is a flow line of $\mathbf{F}$.
  (b) Is $\mathbf{F}$ a gradient vector field? Justify your answer.
  (c) Is the fluid expanding or compressing as it moves? Justify your answer.

There will also be True/False questions on the final exam.