

morse Theory , math 232

Lecture Notes

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Presentations (not included
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- Erman Cineli:
Morse-Novikov Theory
- John Palais:
Lagrangian Floer Theory
- Elijah Fender:
Equivariant Morse Theory

