Mathematics 19B; Winter 2002; V. Ginzburg
Practice Final

1. For each of the ten questions below, state whether the assertion is *true* or *false*:
   (a) To evaluate $\int \sqrt{a^2 - x^2}\, dx$ one should use the trigonometric substitution $x = a \sin\theta$ with $-\pi/2 \le \theta \le \pi/2$.
   (b) $\int \sinh x\, dx = -\cosh x + C$.
   (c) If $a_n \to 0$, then the series $\sum_{n=1}^{\infty} a_n$ converges.
   (d) $\int_0^1 \frac{dx}{x^2}$ converges.
   (e) To find the integral $\int x^3 e^x\, dx$ one should apply the method of integration by parts.
   (f) Assume that $\lim_{n\to\infty} \sqrt[n]{|a_n|} = L$ exists and $L \le 1$. Then the series $\sum_{n=1}^{\infty} a_n$ converges absolutely.
   (g) Assume that b_n is a decreasing sequence, $b_n > 0$ for all n, and $\lim_{n\to\infty} b_n = 0$. Then the series $\sum_{n=1}^{\infty} (-1)^n b_n$ converges.
   (h) $\frac{1}{1-x} = 1 + x + x^2 + x^3 + \dots$ for all $x \ne 0$.
   (i) $\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$ for all x.
   (j) Let a_n be a bounded monotonic sequence. Then a_n converges.

2. Evaluate the following indefinite integrals:
   (a) $\int \frac{x}{x^2 - 5x + 6}\, dx$,
   (b) $\int x\, e^{2x+1}\, dx$,
   (c) $\int \tan^3 x\, dx$.

3. Evaluate the following definite integrals:
   (a) $\int_0^3 \frac{x}{\sqrt{x^2+16}}\, dx$,
   (b) $\int_0^{\frac{\pi}{2}} \cos^6 x\, \sin^3 x\, dx$,
   (c) $\int_0^{\pi^2} \sin\sqrt{x}\, dx$.

4. Find the volume of the solid obtained by rotating the region bounded by $y = \sqrt{4 - x}$, $x = 0$, $y = 0$ about the x-axis.

5. Evaluate the following improper integral: $\int_1^{\infty} \frac{\ln x}{x^2}\, dx$.

6. Test the following series for convergence, absolute convergence, or divergence:

   (a) $\sum_{n=2}^{\infty} \frac{(-1)^n}{n(\ln n)^3}$,

   (b) $\sum_{n=0}^{\infty} (-1)^n \sin n$,

   (c) $\sum_{n=1}^{\infty} \left(\frac{n}{n+1}\right)^{n^2}$

7. Find the Maclaurin series of the function $y = \frac{1}{\sqrt{1-x}}$ and determine its radius of convergence.