Mathematics 19B; Winter 2002; V. Ginzburg
Practice Midterm I

1. For each of the ten questions below, state whether the assertion is *true* or *false*:

   (a) $\int \cos x\, dx = \sin x + C$.

   (b) $\int_2^7 \cos 3x\, dx = \frac{1}{3}(\sin 3x + 13)|_2^7$.

   (c) $\int_a^b f(g(x))g'(x)\, dx = \int_a^b f(u)\, du$.

   (d) The area of the region bounded by the curves $y = x^3 - 14x^2 - 8x + 4$, $y = 6x^2 - 5$, $x = 3$, and $x = 7$ is $-\frac{143}{12}$.

   (e) If $\int_a^b f(x)\, dx$ is negative, then $f(x_*) < 0$ at some point $a \leq x_* \leq b$.

   (f) $\frac{d}{dx} \int_0^x f(t)\, dt = f(x)$, provided that $y = f(x)$ is a continuous function.

   (g) $\lim_{n\to\infty} \frac{1}{n} \sum_{j=1}^n \sin\left(\frac{\pi j}{n}\right) = \int_0^\pi \sin x\, dx$.

   (h) $\lim_{n\to\infty} \frac{1}{n} \sum_{j=1}^n \left(1 + \frac{j}{n}\right)^2 = \int_0^1 x^2\, dx$.

   (i) Let $f(t)$ be the rate of growth of a population. Then $\int_a^b f(t)\, dt = F(b) - F(a)$, where $F(t)$ is the population as a function of time.

   (j) $\int_0^x \sin t\, dt = -\cos x$.

2. Evaluate the following indefinite integrals:

   (a) $\int \cos\left(x + \frac{\pi}{2}\right) dx$,

   (b) $\int e^{3x} dx$,

   (c) $\int \frac{x}{x^2+2} dx$.

3. Evaluate the following definite integrals:

   (a) $\int_0^1 \left(y^3 - 1\right)^2 y^2 dy$,

   (b) $\int_1^0 \frac{1}{3-x} dx$,

   (c) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos\left(x + \frac{\pi}{2}\right) dx$.

4. Evaluate the following derivatives:

   (a) $\frac{d}{dx} \int_x^1 t^3 \ln t\, dt$,

   (b) $\frac{d}{dx} \int_1^{x^3} t^2 \cos t\, dt$.

5. Sketch the region bounded by the curves $y = \sin x$ and $y = -x$, $x = 0$, $x = \pi$, and compute the area of the region.