Mathematics 19A; Fall 2001; V. Ginzburg
Practice Midterm I

1. For each of the ten questions below, state whether the assertion is *true* or *false*.

   (a) The function $f(x) = \sqrt[3]{x}$ is continuous at 0.

   (b) $\lim_{x\to 0} x\sin(\frac{1}{x}) = 0$.

   (c) Let $f(x)$ and $g(x)$ be continuous at $x = a$. Then the function $f(x)g(x)$ is continuous at $x = a$.

   (d) $\lim_{x\to -\infty}(x^7 + 2x) = \infty$.

   (e) Let $f(x) = x^3 - x$. Then the equation $f(x) = 1$ has a solution on the interval $(0, 2)$.

   (f) $\lim_{t\to -\infty} e^t = 0$.

   (g) Assume that $f(x) \geq 0$ and $\lim_{x\to a} f(x) = L$. Then $\lim_{x\to a}\sqrt{f(x)} = \sqrt{L}$.

   (h) $\lim_{t\to 0}\frac{1}{t} = \infty$.

   (i) The function
   $$f(x) = \frac{x+1}{|x|+1}$$
   has only one horizontal asymptote $y = 1$.

   (j) The slope of the tangent to the graph of a function $f(x)$ at the point $P(a, f(a))$ is
   $$m = \lim_{x\to a}\frac{f(x) - f(a)}{x - a}.$$

   (k) The function
   $$f(x) = \begin{cases} x^2 + 2 & \text{for } x \leq 1, \\ 7x - 4 & \text{for } x > 1. \end{cases}$$
   is continuous at $x = 1$.

2. Let
   $$f(x) = \begin{cases} -x & \text{for } x \leq 0, \\ 2x - 1 & \text{for } 0 < x < 1, \\ 1 & \text{for } x \geq 1. \end{cases}$$

   (a) Sketch the graph of $f(x)$ accurately using the solid/open circles where appropriate.

(b) From the graph of $f(x)$ find the following limits. (If the limit does not exist, indicate so.)

i. $\lim_{x\to 0-} f(x)$.
ii. $\lim_{x\to 0+} f(x)$.
iii. $\lim_{x\to 0} f(x)$.
iv. $\lim_{x\to 1-} f(x)$.
v. $\lim_{x\to 1+} f(x)$.
vi. $\lim_{x\to 1} f(x)$.

3. Evaluate the following limits

(a) $\lim_{x\to\sqrt{3}} \frac{x^4-9}{x^2-3}$.

(b) $\lim_{t\to 0} \frac{\sqrt{2-t}-\sqrt{2}}{t}$.

(c) $\lim_{x\to\infty} \frac{3x^3+x-5}{4x^3+x^2+6}$.

(d) $\lim_{x\to\infty}(\sqrt{9x^2+x+2}-3x)$.

4. Find all horizontal and vertical asymptotes of the function

$$f(x) = \frac{x^2+4}{x^2-1}.$$

5. Find the equation of the tangent to the graph of the function

$$f(x) = \frac{x}{1+2x}$$

at the point $P(1, 1/3)$.