Mathematics 19A; Fall 2001; V. Ginzburg
Practice Final

1. For each of the ten questions below, state whether the assertion is *true* or *false*.

   (a) Let $f(x)$ be continuous at $x = a$. Then $\lim_{x\to a} f(x) = f(a)$.

   (b) Let f be a differentiable function and $f'(c) = 0$. Then $f(x)$ necessarily has a local maximum or a local minimum at $x = c$.

   (c) Let $f(x) = a^x$. Then $f'(x) = xa^{x-1}$.

   (d) Let $f(x) = \ln|x|$ Then $f'(x) = 1/x$.

   (e)
   $$\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)},$$
   provided that the limits exist and $\lim_{x\to a} g(x) \neq 0$.

   (f) Assume that $f'(c) = 0$ and $f''(c) > 0$. Then $y = f(x)$ has a local maximum at $x = c$.

   (g) The function
   $$f(x) = \begin{cases} x - 2 & \text{for } x < -1, \\ x^2 - 4 & \text{for } x \geq -1. \end{cases}$$
   is continuous at $x = -1$.

   (h) The function $f(x) = \sqrt{|x|}$ is differentiable at $x = 0$.

   (i) Assume that $f(x)$ is continuous on $[a, b]$ and differentiable on (a, b) and $f(a) = f(b)$. Then there exists a number c in (a, b) such that $f'(c) = 0$.

   (j) Let $f(x)$ and $g(x)$ be differentiable functions and $g'(a) \neq 0$. Then, by L'Hospital rule, one necessarily has that
   $$\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}.$$

2. Find the following limits

   (a) $\lim_{t\to 0} \frac{\sqrt{t+9}-3}{t}$.

   (b) $\lim_{x\to\infty} \frac{\ln x}{\sqrt[3]{x}}$.

   (c) $\lim_{x\to 0} \frac{1-\cos x}{x^2}$.

   (d) $\lim_{x\to 1-} (1 - x) \tanh^{-1} x$.

3. Find $f'(x)$ for the following functions.

   (a) $f(x) = \frac{x^2}{1+x^2}$.

(b) $f(x) = \sin\left(\frac{\ln x}{x}\right)$.

(c) $f(x) = \frac{(x+1)^2}{\sqrt{x^2+2x}}$.

(d) $f(x) = x^{-1}\tan^{-1} x^2$.

4. Find the equation of the tangent line to the curve $x^2 + xy - y^2 = 1$ at the point $(2, 3)$.

5. Let $f(x) = 2x^3 + 3x^2 - 12x + 7$.

   (a) Find $f'(x)$ and $f''(x)$.

   (b) Find the local maxima and minima of f.

   (c) Find the intervals of increase and decrease for f.

   (d) Find the inflection points of f.

   (e) Find the intervals of concavity of f.

6. Find the absolute maximum and the absolute minimum of $f(x) = xe^{-x^2/2}$ on $[0, 2]$.

7. Find the area of the largest rectangle that can be inscribed in a right triangle with legs of length a and b if two sides of the rectangle lie along the legs.