Possible Topics for Senior Seminar

Space filling Hilbert curves

Gauss Bonet Theorem

Kepler's Sphere Packing Conjecture

Mathematical Finance

Special Relativity

Continuous Nowhere Differentiable Functions and other pathological functions

Constructible Polygons and Fermat Primes

Continued Fractions

Cycloid; early quadrature, rectification, isochrone, least time curve

Differential forms in Vector Calculus

Euler's Formula for Polyhedra $V-E+F=2$, Eulers proof...

Splines , Bezier Curves

Fluid Dynamics of Sports Balls

Quaternions

Five Squarable Lunes

Gauss Proof of Fundamental Theorem of Algebra

Early Chinese or Indian Pi computations

Elliptic Functions, Early Results, Lemniscate

Algebraic/ Transcendental numbers, Liouville

Transcendence of π or e. (hard)

Bernoulli numbers and the sums $\sum_{k=0}^{n} k^q$

Euler and the sums $\sum_{k=1}^{\infty} \frac{1}{k^{2n}}$

Unsolvability of certain geometric problems with compass and ruler

Eulers number γ (gamma)

Calculus of Variations, Derivation of the Euler-Lagrange equations, examples

Euler-Maclaurin summation formula

Stirlings formula

Knot theory

Heat equation/Fourier series

Sophie Germain, work on Fermat's theorem

Axiom of choice, survey

Baire Category theorem, nowhere dense sets

Collatz Conjecture

Fermat's early integration methods

Archimedes and the spiral

Archimedes Method and the surface area of the sphere

Cantor type sets, related functions

Four line problem of Pappus, solutions by Fermat and Descartes, analytic geom..

Relationships of Math and Music

Turing machines and the halting time problem

Isoperimetric Problem

Jordan Curve Theorem

Early use of infinitesimals; Cavalieri, Leibniz, Euler

2

1. Algebra

Impossibility of classical ruler and compass constructions (trisecting an angle, doubling the cube)

Fundamental theorem of algebra and the algebraic closure of $\mathbb{C}$

2. Combinatorics

Lattices and sphere-packing

Binary coding theory

Golay codes

Finite projective planes

Graph theory

3. Complex analysis

Gamma function

Riemann's zeta-function: Euler product and functional equation

Möbius transformations and/or Riemann mapping theorem

Conformal mappings

Elliptic (doubly periodic) functions

4. Differential equations

Second order Linear differential equations in the complex domain

Hypergeometric equation

Linear differential equations with regular-singular points

Schwarzian derivative and applications

5. Group theory.

Free groups

Solvable groups

Finitely generated abelian groups

Matrix groups

Representations of finite groups

The Platonic solids and their symmetries

Permutation groups and simplicity of $A_n (n \geq 5)$

Burnside problem

6. Linear algebra

Rational and Jordan canonical forms

Inner product spaces and unitary operators

Real and complex bilinear forms

7. Number theory.

Bernoulli numbers and/or Euler's formula for $\zeta(2n)$

Theory of prime numbers and Tchebychev's theorem

Integers which are sums of two squares

Theory of partitions

Transcendence of e

Transcendence of π

Irrational numbers

Continued fractions

Fermat's last theorem

8. Others

Lemniscatic integrals elliptic integrals

Hyperbolic geometry

Examples of topics suited for the Seminar

1. Algebraic and transcendental numbers. Liouville numbers.
2. Construction of Peano curves.
3. Euler's formula for polyhedra, and the regular Platonic bodies.
4. Bernoulli numbers and the sums $S_p(n) := \sum_{k=0}^{n-1} k^p$.
5. The Pentagon, the Golden Ratio, and the discovery of irrational numbers.
6. Fibonacci numbers.
7. Unsolvability of certain geometric problems by compass and ruler: Doubling of the cube, trisection of the angle, the regular heptagon.
8. Transcendence of $e := \sum_{n=0}^{\infty} \frac{1}{n!}$.
9. Transcendence of π.
10. Conditionally convergent series: Riemann's theorem on the rearrangement of terms.
11. Some proofs of the theorem of Pythagoras (a) in the plane (b) in Hilbert space.
12. Properties of the Gamma function.
13. Riemann's Zeta function: Computation and transcendence of the numbers $\zeta(2n)$ for $n \in \mathbb{N}$.
14. Apery's theorem: $\zeta(3)$ is irrational.
15. Involutes, Evolutes, and the Tautochrone.
16. The Brachistochrone.
17. Calculus of Variations: Derivation of the Euler-Lagrange equations; examples.
18. Legendre transformation: Equivalence of Hamilton's equations with the Euler-Lagrange equations.
19. Cross ratios, and the description of Moebius transformations in the complex plane.
20. Models of non-Euclidean geometries using Moebius transformations.
21. The isoperimetric problem.
22. Steiner's problem and its generalization.
23. The prime number theorem: $A(x) \sim \frac{x}{\log x}$.
24. Lipschitz continuity of convex functions $f : \Omega \to \mathbb{R}$ on open sets $\Omega' \subset\subset \Omega \subset \mathbb{R}^n$.
25. Computation of multiple integrals using the theorem of residues.